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A Simple Power Spectral Analysis for Heart Rate Variability

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1. Introduction

Fourier transform shows important information of a time series data in the frequency domain. Practical data sets are inevitably limited in length because we cannot observe data sets for an infinite time and data sets are also usually obtained only at a set of *N* discrete times t_k within a total data length *T*. The reciprocal of the data length *T* defines the frequency resolution and the reciprocal of the data interval, the sampling frequency, limits the maximum frequency to be analyzed. Fourier series expansion defined in Eq.1 is the sum of the trigonometric functions with the coefficients a_n and b_n given by Eq.2 (e.g.[1])

$$x(k\Delta t) = \sum_{n=0}^{N} [a_n \cos(n\omega_0 k\Delta t) + b_n \sin(n\omega_0 k\Delta t)] \quad (1)$$

$$a_n = \frac{2}{M}\sum_{k=0}^{M} x(k\Delta t)\cos(n\omega_0 k\Delta t)$$
$$b_n = \frac{2}{M}\sum_{k=0}^{M} x(k\Delta t)\sin(n\omega_0 k\Delta t) \quad (2)$$

where ω_0= 1/*T*, *n* is the order of harmonics, Δt is the sampling interval, and k is a positive integer.

Δt is required to be constant to keep the orthogonality between cross terms so that the cross term summation equals to zero. The present method neither requires Δt to be constant $(\Delta t_i \neq \Delta t_j)$ nor the orthogonality between the cross terms.

R wave of ECG of adults repeats almost once a second but it fluctuates which is called physiological arrhythmia. The interval of R wave from the successive R waves is called RR interval and it is considered to be the summation of sinusoids. Thus, the RR interval is often analyzed in a frequency domain by a power spectrum. The high frequency ranges, HF (e.g. 0.2-0.4 Hz), is said to reflect parasympathetic nerve activation and the low frequency ranges, LF (e.g. 0.05-0.2 Hz), is said to reflect both the sympathetic and parasympathetic nerve activations. LF/HF is said to reflect sympathetic nerve activation.

2. Method

Eq. 1 can be substituted by an unequally spaced Fourier expansion defined by Eq. 3 in which $e(t_k)$ is an error and Δt varies by the positive integer k. Using a vector-matrix notation, Eq. 3 can be rewritten as Eq. 4 for *N* time series data. In order to minimize *e* in the least square sense, an optimal Θ for minimizing J, $J = \mathbf{e}^{\mathrm{T}}\mathbf{e} = (\mathbf{x} - \Omega\boldsymbol{\Theta})^{\mathrm{T}}(\mathbf{x} - \Omega\boldsymbol{\Theta})$, is calculated by differentiating J with respect to $\boldsymbol{\Theta}$ and equating it to zero yielding Eq.5

$$x(t_k) = \sum_{n=0}^{N} [a_n \cos(n\omega_0 t_k) + b_n \sin(n\omega_0 t_k)] + e(t_k) \quad (3)$$

$$\mathbf{x} = \Omega\boldsymbol{\Theta} + \mathbf{e} \quad (4)$$

where $\mathbf{x}$ is a time series vector, Ω is the data matrix, $\boldsymbol{\Theta}$ is the parameter vector, and $\boldsymbol{e}$ is the error vector.

$$\boldsymbol{\Theta} = (\Omega^{\mathrm{T}}\Omega)^{-1}\Omega^{\mathrm{T}}\mathbf{x} \quad (5)$$

The algorithm described above is called regression analysis and the coefficients a_n and b_n are called regression coefficients. Once we have the regression coefficients, we can estimate their standard deviations which are given by Eq.6. (e.g. [2])

$$\sigma_n = \sqrt{(\mathbf{e}^{\mathrm{T}}\mathbf{e}/Q)\mathrm{diag}(\Omega^{\mathrm{T}}\Omega)_n^{-1}} \quad (6)$$

where *Q* can be calculated as *Q* = *M* - 2*N* – 1 by using *M* and *N* variables defined in Eq.1. The algorithm described above is implemented in Matlab and is applied to real RR intervals.

For the experiment, a subject was asked to sit down on a chair for about 10 minutes. Electrodes for ECG were placed on the CM5 position of her chest. By detecting the peaks of R waves, the corresponding R waves' time occurrence was measured. The RR intervals were calculated by subtracting successive R wave occurrence times.

3. Results and Discussions

RR intervals were calculated from a 625 s ECG recording. The power spectrum and its 95% confidence intervals of the recorded signal were also calculated by Eqs.(5) and (6), respectively. The results are shown in Fig. 1.

Upon spline interpolation and evenly spaced sampling, one obtains a confidence interval with the conventional FFT by dividing a signal into several pieces, computing the FFTs of the pieces, and calculating the means and the standard deviations of the pieces at every frequency. However, the division of the recording signal into pieces has the drawback of reducing the frequency resolution of the signal since the resolution is given by the reciprocal of the length of a piece. This issue is fatal when we analyze very low frequency components.

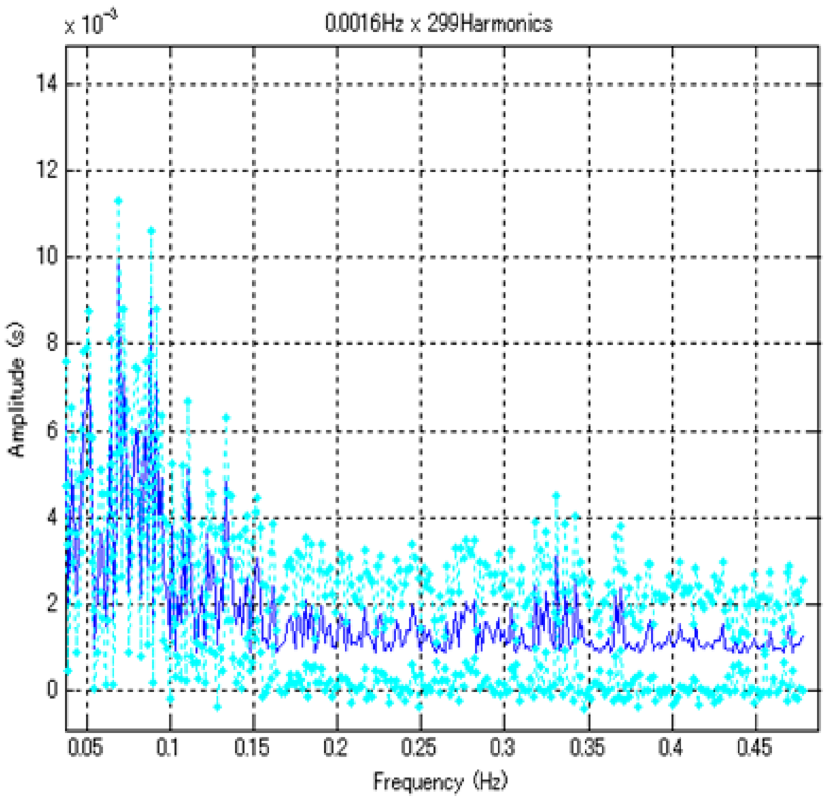

Fig. 1 The solid line is the Power Spectrum of RR intervals and the dashed line is the 95% interval of the RR intervals

As an example, the heart rate variability of a 625 second long measured signal is shown in Fig.1 having a frequency resolution of 1/625=0.0016 Hz. In Fig.1, up to the 299th harmonics (0.4784 Hz = 299/625 s) of the signal is analyzed.

4. Conclusions

We developed a simple Fourier transformation method for an unequally spaced sampling ECG. The method gives the amplitude and phase distributions and their confident intervals. Conventionally, it is required to divide a time series into several portions before Fourier transformation followed by calculating the confident intervals from the average of the Fourier coefficients. However, the division of the time series means the shortening of the observed time and the reduction of the frequency resolution, consequently. This is a very critical drawback in medical or biological fields due to the difficulty of recording long term (weakly) stationary signal in the field.

Our present method does not require dividing the time series and it gives the best time resolutions. Finally, we obtained the power spectrum of heart rate variability in which the frequency range of 0.05 to 0.15 Hz (30th to 90th harmonics) was dominant. The frequency range of 0.3 to 0.35 Hz reflecting parasympathetic nerve activation was also observed. Although the conventional FFT after spline interpolation yielded closely similar results to the developed method in the high frequency range, it failed in the low frequency range because of the limiting frequency resolution in the conventional method.

References

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